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EXPERIMENTAL–THEORETICAL TECHNIQUE FOR DETERMINING THE EQUATION-OF-STATE PARAMETERS FOR SOILS

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This paper considers an experimental–theoretical technique for determining pressure–strain curves and curves of yield strength versus pressure for soils based on disagreement between data of physical and numerical modeling of wave processes in a system of split Hopkinson bars with a soil sample in a holder. A convergent iterative procedure was developed to refine the equation-of-state parameters of soils obtained by the Kolsky method. The error of the technique is analyzed, the role of friction and the coefficient of sliding friction between the soil and the deformed holder.

Key words: *soils, equation of state.*

1. The Kolsky method [1] for dynamic tests of materials in a system of split Hopkinson bars (SHB) has been the subject of extensive studies (see [2] for a survey of them). Modifications of the method for testing the properties of soils and other loose media are directed toward determining both longitudinal and radial stresses in samples. This is done using either an additional measuring bar butted perpendicular to the soil sample or a confining holder [3, 4]. The use of an elastic holder allows one to ignore radial strains of the soil sample and to assume that in the absence of friction, its stress–strain state corresponds to uniaxial strain.

In experimental studies of the dynamic properties of materials using the Kolsky method, the strains $\varepsilon_z(t)$ and stresses $\sigma_z(t)$ in the examined soil sample are determined from the longitudinal strain pulses $\varepsilon_I(t)$, $\varepsilon_{\rm r}(t)$, and $\varepsilon_{\rm y}(t)$ in the measuring bars (the dependence of the quantities on time t is omitted) (see [1]):

$$\varepsilon_z = \frac{c}{l_0} \int\limits_0^t (\varepsilon_I - \varepsilon_{\rm r} - \varepsilon_{\rm y})\, dt; \tag{1}$$

$$\sigma_z = 0.5E(\varepsilon_I + \varepsilon_{\rm r} + \varepsilon_{\rm y}). \tag{2}$$

Here the z axis of the cylindrical coordinate system is directed along the axis of the measuring bars and the r axis is perpendicular to it. The length of the soil sample is denoted by l_0; c is the wave propagation velocity in the measuring bars; ε_I is the loading pulse; $\varepsilon_{\rm r}$ is the reflected axial strain pulse in the loading bar; and $\varepsilon_{\rm y}$ is the transmitted strain pulse recorded in the support bar. Eliminating the time, we obtain the Hugoniot adiabat $\sigma_z(\varepsilon_z)$. The radial stresses $\sigma_{\rm r}(t)$ in the soil sample are determined from the circumferential strains $\varepsilon_\theta(t)$ on the inner surface of the holder [4]:

$$\sigma_{\rm r} = -E_J \,\frac{d_2^2 - d_1^2}{2d_1^2}\, \varepsilon_\theta. \tag{3}$$

Here E_J is the Young's modulus of the holder material and d_1 and d_2 are the inner and outer diameters of the holder, respectively. Relation (3) is derived using the well-known solution of the problem of loading of a thick-walled tube by internal pressure. The strain state of the holder is considered homogeneous.

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If the soil sample l_0 and the holder L have different lengths, more exact results are obtained using formula [5]

$$\sigma_\mathrm{r} = -E_J \frac{d_2^2 - d_1^2}{2d_1^2} \frac{L}{l_0} \varepsilon_\theta. \tag{3a}$$

For large strains of soil samples (20–30%), one can also take into account the change in the loading region of the holder during compression of the soil sample

$$\sigma_\mathrm{r} = -E_J \frac{d_2^2 - d_1^2}{2d_1^2} \frac{L}{l} \varepsilon_\theta, \tag{3b}$$

where $l(t) = l_0(1 + \varepsilon_z(t))$ is the current length of the soil sample and $\varepsilon_z(t)$ is determined from formula (1). From the condition of a uniaxial strain state, we obtain the pressure

$$p = -(\sigma_z + 2\sigma_\mathrm{r})/3. \tag{4}$$

For plastic compression of the soil, the radial component of the stresses tensor deviator is linked to the yield strength σ_y by the relation $s_\mathrm{r} = \sigma_\mathrm{r} + p = \sigma_\mathrm{y}/3$, and, hence,

$$\sigma_\mathrm{y} = 3(\sigma_\mathrm{r} + p). \tag{5}$$

Thus, relations (1)–(5), used in the modified Kolsky method [4] define the Hugoniot adiabat $\sigma_z(\varepsilon_z)$, the hydrostatic pressure $p(\varepsilon_z)$, and the dependence of the yield strength on the pressure in the soil. The obtained relations need to be verified since at large compression ratios, the difference in length between the holder and the soil sample becomes significant. The assumption of homogeneity of the strain state of the holder is no longer valid, and the error of the radial stress can increase.

The measurement errors of radial and longitudinal stresses in soil samples under small strains (up to 10%) were analyzed previously [5, 6] using the Dinamika-2 software [7], which is based on a variational-difference method of second-order approximation in space and time. Numerous calculations and their comparison with experimental data show that the software provides for sufficiently accurate (for applications) calculations of the propagation and interaction of waves in the bars and holder.

2. The iterative algorithm for numerical refinement of the modified Kolsky method at large stresses and strains in soils consists of the following. Relations (1)–(5) are applied to experimental strain pulses in the bars and holder to obtain the required equation-of-state parameters. These parameters are used in numerical calculations (calculation No. 1), and the calculated strain pulses $\varepsilon_\mathrm{r}^{(1)}(t)$ and $\varepsilon_\mathrm{y}^{(1)}(t)$ in the measuring bars and $\varepsilon_\theta^{(1)}(t)$ in the holder are compared with experimental strain pulses $\varepsilon_\mathrm{r}^0(t)$, $\varepsilon_\mathrm{y}^0(t)$, and $\varepsilon_\theta^0(t)$. The subscript (1) indicates the values obtained in numerical calculation No. 1. It is assumed that if the calculated strain profiles in the bars and the holder agree with the experimental data, the chosen model parameters of the soil will also be close to the true ones. In the case of a large difference between the strain pulses, the model parameters are corrected in proportion to the difference using the iterative procedure.

Experimental data, as a rule, are shown as a table of strain pulses versus time with a certain step; calculation data are also obtained with the same step. The difference between the pulses is evaluated by the formulas

$$\max\left(|\varepsilon_\alpha^0(t_i) - \varepsilon_\alpha^{(1)}(t_i)|/|\varepsilon_\alpha^0(t_i)|\right) \cdot 100\% < \delta_\varepsilon, \tag{6}$$

where $i = \overline{1, N}$ (N is the dimension of the value tables), $\alpha = \{R, T, \theta\}$, and δ_ε is the specified relative error equal to the measurement error (about 5%). The convergence of the iterative process is controlled by comparing the equation-of-state parameters of the soil obtained in the $(k+1)$th and kth steps:

$$(|p^{(k+1)} - p^{(k)}|/|p^{(k)}|) \cdot 100\% < \delta, \qquad (|\sigma_\mathrm{y}^{(k+1)} - \sigma_\mathrm{y}^{(k)}|/|\sigma_\mathrm{t}^{(k)}|) \cdot 100\% < \delta. \tag{7}$$

Here δ is the specified relative error and $p(\varepsilon)$ and $\sigma_\mathrm{y}(p)$ are functions that give the least squares approximations of the pressure (4) and the yield strength (5), respectively. The particular form of the functions p and σ_y depends on the chosen model of soil deformation.

The procedure of refining the equation-of-state parameters consists of the following. An increase (a decrease) in the yield strength (5) at constant pressure (4) results in an increase (a decrease) in the radial stress $\sigma_\mathrm{r}(t)$ in the soil. The circumferential strain in the elastic holder changes in proportion to $\sigma_\mathrm{r}(t)$ [see formula (3b)]:

$$\sigma_\mathrm{r}/\varepsilon_\theta^0 = \sigma_\mathrm{r}^{(1)}/\varepsilon_\theta^{(1)}.$$

Evaluating $\sigma_\mathrm{r}(t)$ and using formulas (4) and (5), we determine the pressure $p = -(\sigma_z^{(1)} + 2\sigma_\mathrm{r})/3$ and the yield strength

$$\sigma_\mathrm{y}(p) = 3(p + \sigma_\mathrm{r}). \tag{8}$$

The strain pulses in the measuring bars and the holder $\{\varepsilon_{\rm r}^{(2)}(t), \varepsilon_{\rm y}^{(2)}(t), \varepsilon_\theta^{(2)}(t)\}$ obtained in the next numerical calculation (calculation No. 2) are compared with experimental strain pulses $\{\varepsilon_{\rm r}^0(t), \varepsilon_{\rm y}^0(t), \varepsilon_\theta^0(t)\}$, respectively.

If condition (6) is not satisfied, the pressure curve is corrected with the unchanged pressure dependence of the yield strength. We assume that the ratio of the longitudinal strain in the soil sample $\varepsilon_z^{(2)}$ to the strain determined from formula (1) is equal to the ratio of the same values in full-scale and numerical experiments:

$$\varepsilon_z\Big(\frac{c}{l_0}\int\limits_0^t(\varepsilon_I^0-\varepsilon_{\rm r}^0-\varepsilon_{\rm y}^0)\,dt\Big)^{-1}=\varepsilon_z^{(2)}\Big(\frac{c}{l_0}\int\limits_0^t(\varepsilon_I^{(2)}-\varepsilon_{\rm r}^{(2)}-\varepsilon_{\rm y}^{(2)})\,dt\Big)^{-1}.$$

From this, we obtain the axial strains of the soil $\varepsilon_z(t)$ that correspond to the values of $\varepsilon_{\rm r}^0(t)$:

$$\varepsilon_z=\varepsilon_z^{(2)}\int\limits_0^t(\varepsilon_I^0-\varepsilon_{\rm r}^0-\varepsilon_{\rm y}^0)\,dt\Big/\int\limits_0^t(\varepsilon_I^{(2)}-\varepsilon_{\rm r}^{(2)}-\varepsilon_{\rm y}^{(2)})\,dt. \tag{9}$$

The pressure p corresponding to the strain ε_z is determined from the relation

$$p/(\varepsilon_I^0+\varepsilon_{\rm r}^0+\varepsilon_{\rm y}^0)=p^{(2)}/(\varepsilon_I^{(2)}+\varepsilon_{\rm r}^{(2)}+\varepsilon_{\rm y}^{(2)}).$$

Using (2), we have

$$p=p^{(2)}(\sigma_z^0/\sigma_z^{(2)}). \tag{10}$$

Here $p(t)$ is a function of the pressure that corresponds to $\varepsilon_{\rm y}^0(t)$.

If conditions (6) are not satisfied, the procedure of refining the dependence of the yield strength and the pressure curve (8)–(10) is repeated. If conditions (7) are satisfied, the iterative processes is interrupted and the difference between the calculated and experimental strain pulses allows one to estimate the errors of the experimental–theoretical technique and the mathematical model of the medium in the given full-scale experiment.

To eliminate the effect of the experimental error on the convergence of the numerical algorithm, as "experimental data" we take the data of a numerical calculation with specified equation-of-state parameters, which are considered "reference." To describe the deformation of the soil sample in the holder, we employ the Grigoryan soil model [8], which includes pressure–strain and yield strength–pressure nonlinear dependences. The dependences $p(\varepsilon)$ and $\sigma_{\rm y}(p)$ are approximated by [9, 10]

$$p=-\rho_0a^2\varepsilon/(1+b\varepsilon)^2; \tag{11}$$

$$\sigma_{\rm y}=\sigma_0+\mu p/(1+\mu p/(\sigma_{\rm y}^{\max}-\sigma_0)), \tag{12}$$

where a, b, σ_0, μ, and $\sigma_{\rm y}^{\max}$ are constants, $\varepsilon=\rho_0/\rho-1$, and ρ_0 and ρ are the initial and current densities of the soil, respectively. The constant a is the wave propagation velocity in the soil at a density close to the initial one in the absence of shear strength; the dimensionless constant b characterizes the limiting compressibility of the soil. The constants σ_0, μ, and $\sigma_{\rm y}^{\max}$ characterize the cohesion, the angle of internal friction of the soil, and the maximum yield strength, respectively. Approximation (11) for the dependence $p(\varepsilon)$, was obtained earlier for the Hugoniot adiabat of soil at stresses of 0.1–5.0 GPa in plane-wave experiments [9]

$$\sigma=\rho_0V^2\varepsilon/(1+b\varepsilon)^2. \tag{13}$$

Here V is the longitudinal wave velocity in the soil at $\rho=\rho_0$. Relation (13) follows from the linear dependence $D=V+bu$, where D is the shock-wave velocity and u is the mass velocity in the soil. As shown by the experiments of [8, 10, 11], relation (12) for loose soil at stresses of up to 0.1 GPa can also be considered linear ($\sigma_{\rm y}=\mu p$). Under conditions of a quasi-uniaxial strain state, the side-pressure coefficient $K_\sigma=\sigma_{\rm r}/\sigma_z$ is constant and, hence, in the absence of radial displacement of the sample, the relation between σ_z and p (or V a) has the form

$$\sigma_z=-p/(1+2\mu/3) \quad \text{or} \quad V^2=a^2/(1+2\mu/3).$$

For $\mu=1$ and $K_\sigma=0.4$, we have $p=-0.6\sigma_z$ and $V\approx1.3a$. The modification of the Kolsky method [4] using a confining holder allows one to examine the behavior of soils at stresses above 0.1 GPa, at which the applicability of the equation of state (12) has been inadequately studied.

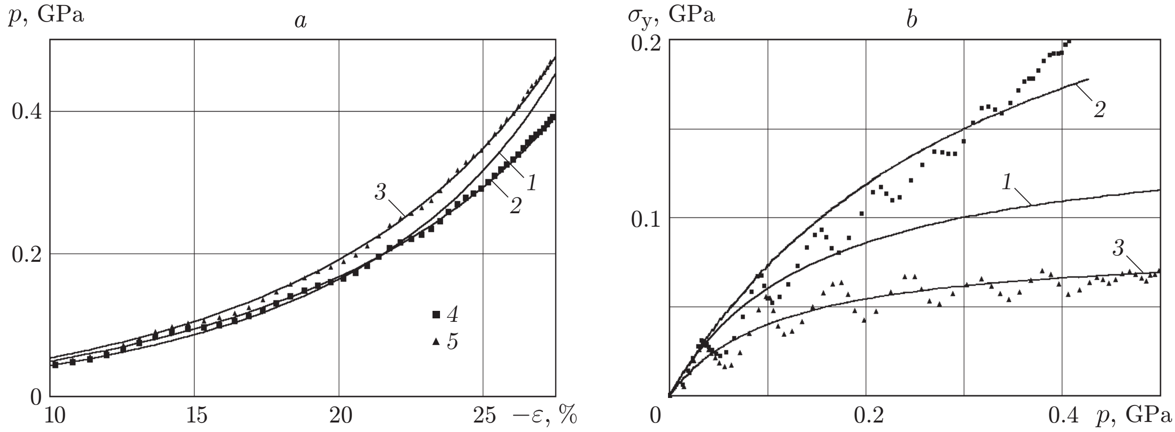

Fig. 1

TABLE 1

No.	a, m/sec	b	μ	$\sigma_{\mathrm{y}}^{\max}$, GPa	Note
1	400	2.2	1.0	0.150	"Reference" values
2	445	1.92	0.95	0.317	Formula (3a)
3	458	2.0	0.75	0.845	Formula (3b)
4	—	—	1.27	0.13	First iteration
5	427	2.1	—	—	Second iteration
6	427	2.1	1.27	0.13	Final result

3. The formulation of the problem of numerical modeling of the deformation of soil samples in a SHB system and in a confining holder is similar to that in [6]. The lengths of the loading bar (I) and support bar (II) are equal to 1000 mm and their radii are $R = 10$ mm. The holder dimensions are $20 \times 40 \times 15$ mm (inner diameter d_1, outer diameter d_2, and length L). The length of the soil sample is $l_0 = 9$ mm. The material of the bars and the holder is steel with a Young's modulus of 200 GPa and a density of 7.8 g/cm^3. The loading pulse $\varepsilon_I^0(t)$ has the shape of a trapezoid with a maximum value of 0.6 GPa, a pulse duration $T = 150$ μm, and a rise time to the maximum value of 15 μsec. A difference grid with square meshes is used. The number of meshes is 5×500 in the bar, 10×9 in the soil sample, and 15×10 in the holder.

In Fig. 1, curves 1 show the initial ("reference") curves of pressure versus volumetric strain (a) and of yield strength versus pressure in the soil (b). Curve 1 in Fig. 1a was obtained for $a = 400$ m/sec, $b = 2.2$, and $\rho_0 = 1.6$ g/cm^3. In Fig. 1b, curve 1 corresponds to relation (12) for $\sigma_0 = 0$, $\mu = 1$, and $\sigma_{\mathrm{y}}^{\max} = 0.15$ GPa. These equation-of-state constants are given in line 1 of Table 1. The strain pulses obtained in the calculation using these constants are denoted, as before by $\varepsilon_{\mathrm{r}}^0$, $\varepsilon_{\mathrm{y}}^0$, and ε_θ^0. Their corresponding pressures calculated by formulas (1)–(4) of the Kolsky method are shown in Fig. 1a by points 4 and 5 [points 4 refer to calculations by formula (3a) and points 5 refer to calculations by formula (3b)]. The obtained discrete pressure values are approximated by relations of the form (11), which are shown in Fig. 1a by curves 2 and 3, respectively. The yield strength of the soil is determined from formulas (1)–(5) and is shown in Fig. 1b by points (notation same as in Fig. 1a). The constants of relations (11) and (12) determined by the least-squares method are given in lines 2 and 3 of Table 1.

In Fig. 2, the dashed and solid curves show the errors of the calculations by the equation of state using relations (3a) and (3b), respectively (with respect to the "reference" values). Curves 1 refer to the dependence $p(\varepsilon)$ and curves 2 refer to $\sigma_{\mathrm{y}}(p)$. The equation-of-state constants for soils at lower pressures are well known [4, 10–12].

An analysis shows that at high pressures, formula (3b) gives a smaller error than formula (3a). The errors of the curve of $p(\varepsilon)$ (solid curve 1 in Fig. 2) reach 20% at moderate pressures and 5–10% at high pressures. The errors of the yield strength $\sigma_{\mathrm{y}}(p)$ (solid curve 2 in Fig. 2) are 30–40%. Thus, the use of the modified Kolsky method at high pressures leads to errors that are 5–6 times higher than the measurement error (6%) [4].

To refine the equation of state for soil obtained by the Kolsky method, we performed numerical calculations of the propagation of a strain pulse ε_I^0 in the SHB system (iteration 1). The equation-of-state constants are given in line 3 of Table 1. Next, we determine the longitudinal strain pulses $\varepsilon_{\mathrm{r}}^{(1)}$ and $\varepsilon_{\mathrm{y}}^{(1)}$ in bars I and II,

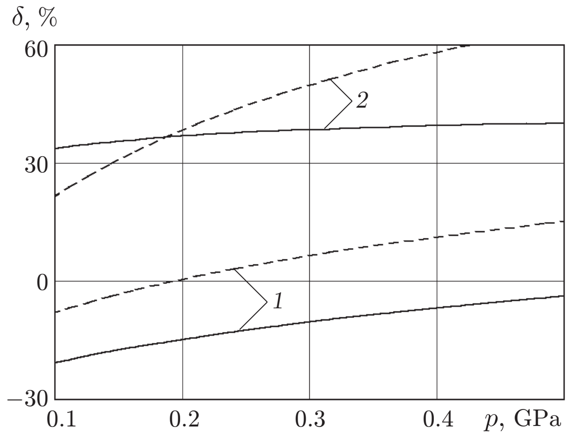

Fig. 2

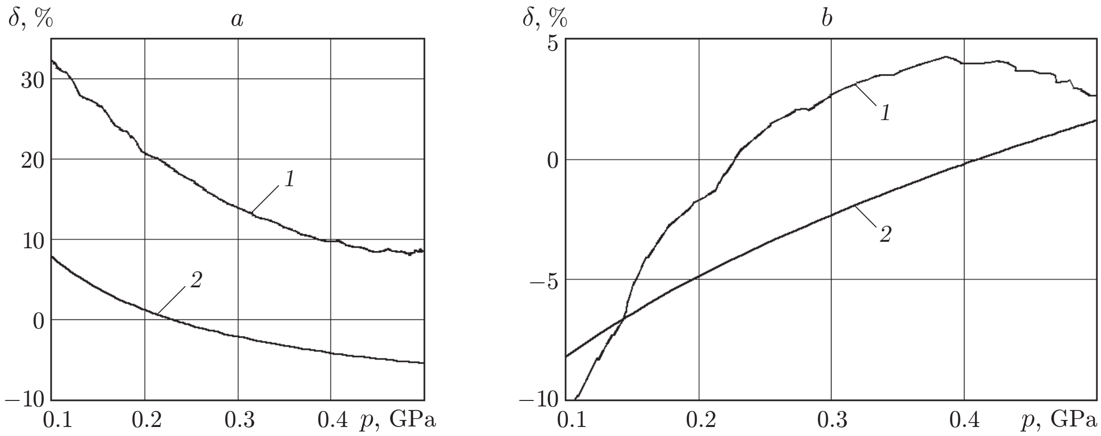

Fig. 3

respectively, and the circumferential strain $\varepsilon_\theta^{(1)}$ in the holder as functions of time. The obtained values are substituted into (8) to determine the dependence $\sigma_{\rm y}(p)$. In Fig. 3a, curve 1 shows the error of the circumferential strain $\delta(\varepsilon_\theta^{(1)}) = ((\varepsilon_\theta^0 - \varepsilon_\theta^{(1)})/\varepsilon_\theta^0) \cdot 100\%$, which varies in the range 10–30%.

The pressure dependence of the yield strength obtained by formula (8) is approximated by curve (12). The obtained constants of the function $\sigma_{\rm y}(p)$ are given in line 4 of Table 1. The relative errors of the dependence $\sigma_{\rm y}(p)$ are shown in Fig. 3a by curve 2. The values of the yield strength corrected by formula (8) differ from the "reference" values by $\pm 5\%$. We note that the error in determining the dependence $\sigma_{\rm y}(p)$ by the Kolsky method (solid curve 2 in Fig. 2) reached 40% for the strain pulses ε_θ^0 and $\varepsilon_\theta^{(1)}$ in the holder varied in the range 10–30% (curve 1 in Fig. 3a).

In iteration 2, we correct the pressure curve with the unchanged parameters μ and $\sigma_{\rm y}^{\max}$ in formula (12) for the yield strength. The equation-of-state parameters used in calculation No. 2 are given in line 4 of Table 1. The calculation results are given in Fig. 3b by curve 1, which corresponds to the relative error of the transmitted strain pulse:

$$\delta(\varepsilon_{\rm y}^{(2)}) = ((\varepsilon_{\rm y}^0 - \varepsilon_{\rm y}^{(2)})/\varepsilon_{\rm y}^0) \cdot 100\%.$$

The error is 5–10%. The relative error of the curve of $p(\varepsilon_z)$ obtained using formulas (9) and (10) with respect to the results of calculation No. 2 is shown by curve 2 in Fig. 3b. Correction of the curve of $p(\varepsilon_z)$ reduces the error from 5–20% (solid curve 1 in Fig. 2) to $\pm 5\%$ (curve 2 in Fig. 3b) over the entire range considered. The constants of relation (11), which approximates $p(\varepsilon_z)$, are given in line 5 of Table 1.

The next iterations do not lead to a considerable refinement of the equation-of-state parameters. In the entire pressure range, the error remains within $\pm 5\%$ with respect to the known "reference" solution. Apparently, this error is an inherent error of the Kolsky method for the chosen equation-of-state parameters, loading-pulse duration T, and geometry of the soil sample.

4. Let us estimate the effect of dry friction. In practice, the pulses recorded in a SHB system can be distorted by friction of the soil on the holder surface and the ends of the bars. The value of the distortion depends not only on the type of soil but also on the holder material. In using the Coulomb–Amonton dry friction hypothesis, as the friction coefficient one can employ the coefficient of proportionality between the axial stress and the maximum shearing stress (the so-called angle of internal friction). This approach to studying wave processes in a holder with a soil is approximate and ignores the contact interaction leading to an inhomogeneous stress–strain state in the soil sample. The homogeneity condition for the stress–strain state of the examined sample implies equality of the forces applied to the sample ends. In the case of equality of the elastic moduli and cross-sectional areas of the loading and support bars, we obtain the well-known condition

$$\varepsilon_I + \varepsilon_{\rm r} = \varepsilon_{\rm y}. \tag{14}$$

The degree of inhomogeneity can also be estimated from the energy conservation law for the SHB system. The work of external forces expended in deformation of the loading bar and the changes in the internal energy in bars I and II are given, respectively, by

$$A = \pi R^2 Ec \int_0^t \varepsilon_I^2\,dt, W_{\rm I} = \pi R^2 Ec \int_0^t \varepsilon_{\rm r}^2\,dt, \qquad W_{\rm II} = \pi R^2 Ec \int_0^t \varepsilon_{\rm y}^2\,dt. \tag{15}$$

We introduce the work of external forces A_g expended in deformation of the soil and holder. Writing the energy balance equation

$$A = W_{\rm I} + A_g + W_{\rm II},$$

we express the quantity A_g in terms of the known pulses in the measuring bars:

$$A_g = \pi R^2 Ec \int_0^t (\varepsilon_I^2 - \varepsilon_{\rm r}^2 - \varepsilon_{\rm y}^2)\,dt. \tag{16}$$

The change in the internal energy of the soil sample under deformation is evaluated as

$$W = 0.5\pi R^2 Ec \int_0^t (\varepsilon_I^2 - (\varepsilon_{\rm r} + \varepsilon_{\rm y})^2)\,dt. \tag{17}$$

Here we use the expression of the mean stress and strain in the soil given by formulas (1) and (2).

The work A_g is written as

$$A_g = W + A_f + W_k, \tag{18}$$

where A_f is the work of friction of the soil on the holder surface and W_k is the kinetic energy of the holder acquired upon the possible rigid-body displacement of the holder due to cohesion with the soil. The elastic strain energy of the holder is easily found to be negligible compared to (17). From Eq. (18), using (16) and (17), we express the dissipative term

$$A_f + W_k = 0.5\pi R^2 Ec \int_0^t (\varepsilon_I^2 - (\varepsilon_{\rm r} - \varepsilon_{\rm y})^2)\,dt. \tag{19}$$

If the conventional homogeneity condition for the stress–strain state of the tested material sample (14) is satisfied, the right side of (19) vanishes, which implies that the work of friction and the kinetic energy of the holder vanish simultaneously.

The work of friction of the soil on the holder surface is evaluated as follows. The normal pressures on the holder are defined by formula (3a) from the data of a sensor of circumferential strain on the outer surface of the holder. The velocities of the ends of the soil sample are calculated by the Kolsky method. Integrating the work of friction by the trapezoid rule, we obtain

$$A_{fr} = k0.5\pi Rc\frac{b^2 - a^2}{a^2}\,\frac{L}{l_0}\,E_J \int_0^t (\varepsilon_I - \varepsilon_{\rm r} + \varepsilon_{\rm y})\varepsilon_\theta l\,dt, \tag{20}$$

where E_J is Young's modulus of the holder material. Formula (20) approximately describes the work of friction for displacement of the soil along the holder surface. Using (19) and (20), we express the friction coefficient

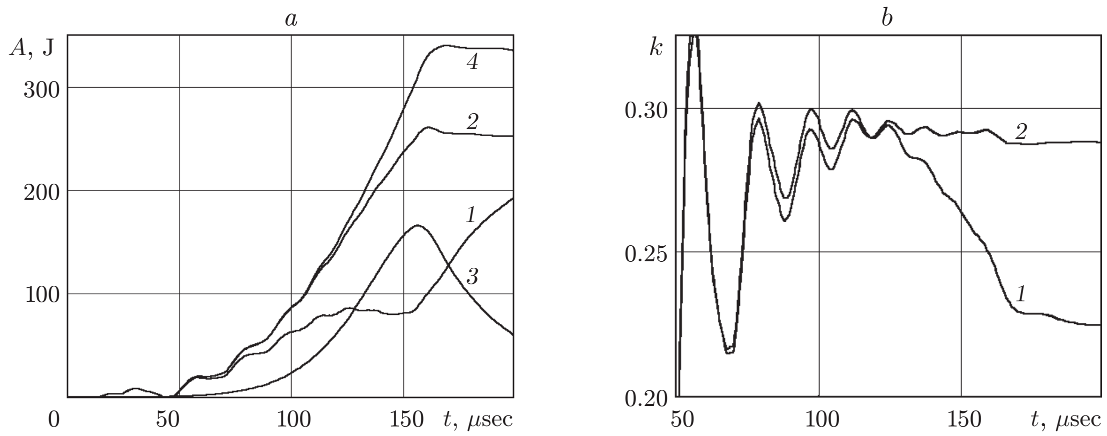

Fig. 4

$$k = \frac{E}{E_J} \frac{l_0}{L} \frac{Ra^2}{b^2 - a^2} \int\limits_0^t (\varepsilon_I^2 - (\varepsilon_{\rm r} - \varepsilon_{\rm y})^2)\, dt \Big/ \int\limits_0^t (\varepsilon_I - \varepsilon_{\rm r} + \varepsilon_{\rm y})\varepsilon_\theta l\, dt. \qquad (21)$$

In calculating the energy by formulas (15)–(18) and using (21), one needs to synchronize the initial strain pulses in the bars and the holder. For this, the reflected pulse front was made coincident with the front of the incident pulse, and the maximum of the mean stress (2) was made coincident with the maximum of the radial stress (3).

To estimate the relations obtained, we performed numerical calculations for loading of a soil sample in a holder for the following versions: 1) an aluminum holder of weight 10 g and $k = 0$ and 0.3; 2) a steel holder of weight 110 g and $k = 0.3$; 3) a steel holder fixed to prevent displacement in the longitudinal direction and $k = 0.3$. The calculations were performed using a contact algorithm with nonpenetration along the normal and sliding along the tangent with dry friction according to Coulomb's law:

$$\dot u'_s = \dot u''_s, \qquad q'_s = -q''_s, \qquad q_s = q'_s = \begin{cases} q_s, & |q_s| \leqslant k|q_\xi|, \\ k|q_\xi| \operatorname{sign}(q_s), & |q_s| > k|q_\xi|. \end{cases}$$

Here $\dot u_\alpha$ and q_α are the components of the displacement velocity and the contact pressure in a local coordinate basis ($\alpha = s, \xi$, where s and ξ are the directions of the tangent and the normal) and k is the coefficient of sliding friction.

In the first version of the calculations for $k = 0.3$, the light aluminum holder started to displace in the loading direction due to cohesion with the soil. In this case, although the relative displacements of the soil and the holder are insignificant, the stresses on the contact surface differ from the stresses in the middle of the soil sample by 15–20%. The disbalance (14) of the strain pulses $\{\varepsilon_I, \varepsilon_{\rm r}, \varepsilon_{\rm y}\}$ is small for both $k = 0$ and $k = 0.3$, which is explained by a local effect of surface friction. The maximum disbalance of the energy $A_f + W_k$ determined form formula (19) is less than 5% of the maximum internal energy W in (17). Consequently, satisfaction of conditions (14) or (19) with an error of 5% is sufficient for integral estimation of the homogeneity of the stress state of the tested soil sample. The quantitative relation between the work of friction and the kinetic energy of the holder depends on the coefficient of sliding friction, the weight of the holder, and other factors.

The use of more massive steel holders in experiments is dictated to the requirement of their elastic deformation in the examined range of stresses (up to 1 GPa). The results of numerical calculations using steel holders are given in Figs. 4 and 5. In Fig. 4a, curve 1 shows the work of friction in the case of free displacement of the holder; curve 2, the energy disbalance calculated by formula (19); curve 3, the kinetic energy of the holder; and curve 4, the work of friction for a fixed holder. Curve 1 in Fig. 4b shows the variation in the coefficient of sliding friction. Its value obtained in the second calculation version (with a free holder) using formula (21) differs from the value $k = 0.3$ specified in the calculations since its calculation ignored the kinetic energy of the holder in (19) and the velocity of holder displacement in (20). In the beginning of the process, a segment is observed on which the friction coefficient oscillates about the specified value (curve 1 in Fig. 4b); next, for the maximum velocity of the holder, the coefficient decreases to 0.25. These changes are related to the oscillatory nature of the processes in the soil sample under reflection of stress waves from the ends of the measuring bars.

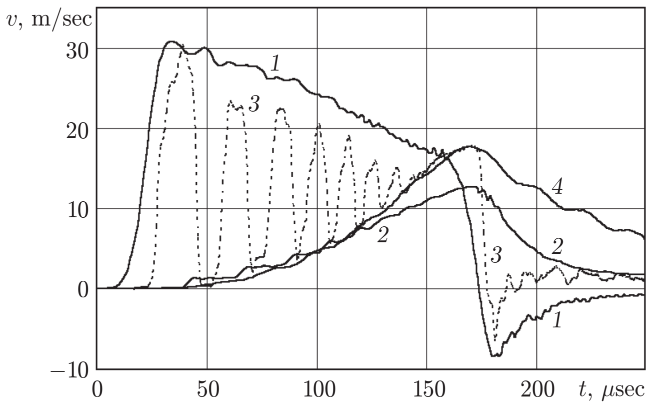

Fig. 5

In Fig. 4b, curve 2 shows the variation in the friction coefficient in the absence of rigid-body displacement of the holder. Its value close to 0.3 is rapidly established, which allows a more accurate estimation of the friction coefficient using formula (21).

Figure 5 illustrates the kinematics of the process in the calculation version with a moving holder. Curves 1 and 2 refer to the velocities of the ends of the soil sample adjoining the loading and support bars, respectively, curve 3 refers to the velocity of displacement of the center of the soil sample, and curve 4 refers to the velocity of the holder. It is evident that in the initial stage, the velocity of the holder is close to velocity of the right end of the soil sample, and in the further process, the holder velocity exceeds the soil velocity. During unloading, the holder is decelerated.

5. Thus, numerical modeling was performed of the wave processes in a SHB system at stresses of up to 1 GPa. By comparison with the "reference" solution, it is shown that the error in determining the dependence $\sigma(\varepsilon)$ for using the Kolsky method does not exceed the experimental error. Because of inaccuracies in determining the stress–strain state of the holder and the side pressures in the soil, the calculation errors for the dependences $p(\varepsilon)$ and $\sigma_{\rm y}(p)$ are much higher. They can be 5–6 times higher than the measurement error of strain pulses in the measuring bars and confining holder, which in practice results in errors of 30–40%. An iterative procedure using the Dinamika-2 software is proposed to refine the parameters of soil models obtained by the Kolsky method. Convergence of the iterations to a known "reference" solution was studied numerically. The stabilized error obtained in this case gives an estimate of the inherent error of the Kolsky method. It depends on the type of soil, loading-pulse duration, and the geometry of the tested sample and holder.

A method is proposed to estimate the work of friction in the holder–soil friction pair from experiments using a SHB system and a holder fixed to prevent its displacement in the longitudinal direction. It is shown that the effect of friction on the strain pulses in the bars decreases considerably for a light holder displaced together with the soil. To correct the equation-of-state parameters taking into account friction, it is possible to use the proposed iterative procedure with a specified friction coefficient.

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REFERENCES

1. G. Kolsky, "Mechanical characteristics of materials at high loading rates," *Mekhanika*, No. 4, 108–119 (1950).
2. J. Zukas, T. Nicholas, H. F. Swift, et al., *Impact Dynamic*, John Wiley (1982).
3. A. M. Bragov, G. M. Grushevsky, and A. K. Lomunov, "Use of the Kolsky method for confined tests of soft soils," *J. Exp. Mech.*, **36**, No. 3, 237–242 (1996).

4. A. M. Bragov, V. P. Gandurin, G. M. Grushevskii, and A. K. Lomunov, "New potentials of the Kolsky method for studying the dynamic properties of soft soils," *J. Appl. Mech. Tech. Phys.*, **36**, No. 3, 476–483 (1995).
5. V. G. Bazhenov, A. M. Bragov, V. L. Kotov, et al., "Analysis of the applicability of a modified Kolsky method for dynamic tests of soils in a deformable casing," *J. Appl. Mech. Tech. Phys.*, **41**, No. 3, 519–525 (2000).
6. G. M. Grushevskii and E. V. Tsvetkova, "Numerical–experimental study of a modified split Hopkinson bar method," in: *Applied Problems of Strength and Plasticity. Methods of Solution* [in Russian], Nizhnii Novgorod Univ., (1992), pp. 116–120.
7. V. G. Bazhenov, S. V. Zefirov, A. V. Kochetkov, et al., "Dinamika-2 software for solving plane and axisymmetric nonlinear problems of nonstationary interaction of structures with compressible media," *Mat. Model.*, **12**, No. 6, 67–72 (2000).
8. S. S. Grigoryan, "Basic concepts of the soil dynamics," *Prikl. Mat. Mekh.*, **24**, No. 6, 1057–1072 (1960).
9. V. A. Lagunov and V. A. Stepanov, "Measurement of the dynamic compressibility of sand at high pressures," *J. Appl. Mech. Tech. Phys.*, No. 1, 88–96 (1963).
10. A. A. Vovk, B. V. Zamyshlyaev, L. S. Evterev, et al., *Behavior of Soils under Pulsed Loading* [in Russian], Naukova Dumka, Kiev (1984).
11. G. V. Rykov, "Experimental study of the stress field for an explosion in a sand soil," *J. Appl. Mech. Tech. Phys.*, No. 1, 85–89 (1964).
12. V. G. Bazhenov, V. L. Kotov, A. V. Kochetkov, et al., "Wave processes in soil upon detonation of a mud cap," *Izv. Ross. Akad. Nauk, Mekh. Zhidk. Gaza*, No. 2, 70–77 (2001).